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Modeling horizontal integration of companies in volatile markets

Modelado de integración horizontal de empresas en mercados volátiles

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ABSTRACT:

Research methods: economic and mathematic modeling, optimization theory, game theory, utility theory, theory of random functions. The following basic results were obtained in the course of research: first of all, it was demonstrated that enterprises have always got more incentives to horizontal integration in the markets characterized by uncertainty of demand or production costs, in case mentioned data is a matter of private information. A joint company can aggregate the information of its departments and streamline production among its business units accordingly. Besides, intensified volatility in the financial markets increases the incentives of enterprises to horizontal integration. Joint ventures may be profitable even in low concentration markets. In this work, we analyze the incentives of enterprises to horizontal integration and the impact the horizontally integrated companies produce on the national welfare in the markets characterized by uncertainty of demand or production costs. The obtained results are compared with the incentives of enterprises to horizontal integration and

RESUMEN:

Métodos de investigación: modelos económicos y matemáticos, teoría de la optimización, teoría de juegos, teoría de la utilidad, teoría de las funciones aleatorias. Los siguientes resultados básicos se obtuvieron en el curso de la investigación: en primer lugar, se demostró que las empresas siempre tienen más incentivos para la integración horizontal en los mercados caracterizados por la incertidumbre de la demanda o los costos de producción, en caso de que los datos mencionados sean una cuestión de privacidad información. Una empresa conjunta puede agregar la información de sus departamentos y optimizar la producción entre sus unidades de negocios en consecuencia. Además, la volatilidad intensificada en los mercados financieros aumenta los incentivos de las empresas a la integración horizontal. Las empresas conjuntas pueden ser rentables incluso en mercados de baja concentración. En este trabajo, analizamos los incentivos de las empresas para la integración horizontal y el impacto que las empresas integradas horizontalmente producen en el bienestar nacional en

the impact the horizontally integrated companies produce on the national welfare in determined markets.
Keywords: economic and mathematic modeling, horizontal integration, regional economics

los mercados caracterizados por la incertidumbre de la demanda o los costos de producción. Los resultados obtenidos se comparan con los incentivos de las empresas para la integración horizontal y el impacto que las empresas integradas horizontalmente producen en el bienestar nacional en mercados determinados.
Palabras clave: modelación económica y matemática, integración horizontal, economía regional

1. Introduction

Relevance of the research topic. The choice of efficient forms and strategies of growth of enterprises' capital and assets determines to a large extent long-term and effective industrial activity, enhancement of competitiveness and provision of sustained high levels of economic development. One of the most popular and important decisions made by the companies refers to the search of effective arrangement forms of consolidation. However, experience suggests that market volatility is a major determinant of integration activity. According to conventional assumptions, enterprises have more incentives to integration in areas (markets) characterized by a higher degree of uncertainty. In this regard, analyzing the incentives of enterprises to horizontal integration and the impact the horizontally integrated companies produce on the national welfare in oligopolistic markets characterized by uncertainty of demand and/or production costs is highly relevant.

The issue of research and optimization of market structures is becoming more and more relevant among the scientists of the world (Qing Yang, Lei Zhang, Xin Wang 2017; Kadiyali, Sudhir & Rao 2001; Kumar, Srivastava, & Singh 2005; Harrigan 1986), the structures in which the company that occupies a downstream position in the technological chain provides the upper companies with input resources and, at the same time, is vertically integrated with one of the upper stream companies. This analysis is still more important from the point of view of assessing the possible anti-competitor effect of those vertically integrated structures as well as the influence of information flows between the downstream and upper-stream enterprises of the area on the incentives to develop the innovations and the National Wealth. The above mentioned problems are especially relevant in case it is required to make the essential information, information on technology, design or specific characteristics of the product in particular, available both to the downstream and upper-stream enterprises of the area. This situation is characteristic of high-technology areas where information exchange regarding downstream and upper-stream products is required in order to provide product compatibility and to avoid additional costs for adjustment and functional improvement. From the antimonopoly point of view, the issue of assessing whether it is feasible or not to prohibit different divisions of a horizontally integrated company to share non-public information received by one of the divisions from third-party sources is really essential.

Enterprises often make long-term decisions without having a clear idea of short-term market conditions. Decisions on horizontal integration are taken in case there is uncertainty of demand and production costs. For example, oil companies can discover more economical (in terms of costs) developments of oil deposits or biotechnological companies can generate new products that will be in great demand. In this context, we can say, the created model assumes that the enterprises study the demand and production costs after they have made the decision on integration. More than that, the enterprises may be to some extent aware of their competitors' demand and production costs. For instance, information on institutional developments, regarding the demand and/or production costs is, as a rule, available to public. On the contrary, the quality of oil deposit development or the results of innovation studies are private information of the companies. Both types of uncertainty are taken into consideration in the model and it is shown that the identified difference plays a fundamental role.

In this work we make an attempt to analyze the incentives of enterprises to horizontal integration and the impact the horizontally integrated companies produce on the national welfare in the markets (Cournot Oligopolies [according to previous studies of Cournot Theory and his followers (Friedman 1982; Amir 1996; Satoh & Tanaka 2016; Shinozaki & Kunizaki 2017)], characterized by uncertainty of demand or production costs. The obtained results are compared with the incentives of enterprises to horizontal integration and the impact the horizontally integrated companies produce on the national welfare in determined markets.

The impact of horizontal integration on the national welfare in the markets characterized by uncertainty also depends on the type of information. In case the uncertainty in the market is private information, horizontally integrated companies can produce a less negative impact on the national welfare than when they act in determined markets. After the integration, the aggregate production output is achieved in a more efficient way, while the divisions characterized by lower production costs produce a relatively larger amount than those characterized by higher production costs. Besides, the variability of the aggregate amount of production goes down, therefore, the national welfare grows. It is established that if the uncertainty is high and the market is very concentrated, horizontally integrated companies lead to growth of national welfare.

If the uncertainty in the area (in the market, for example (Hoskisson & Busenitz 2002)) is private information, horizontally integrated companies are a more frequent phenomenon and are more favorable for the national welfare. In the markets characterized by high volatility, these benefits can compensate for the anti-competitor effects of horizontal integration and, therefore, horizontally integrated companies, in case the uncertainty in the area is private information, increase the national welfare.

The results, in case the uncertainty in the area is private information, can be achieved under the condition that the enterprises receive ideal signals regarding their uncertain and independent characteristics. The enterprises also have more incentives to integration in case these characteristics are interrelated. However, in case these characteristics are correlated to a large extent, the enterprises can have more incentives to integration than in determined markets. Actually, in this case, the enterprises can estimate all the information on their competitors and, therefore, the situation is close the one when the uncertainty in the area (in the market) is public information. Consequently, integration in the markets characterized by uncertainty is more profitable in case the enterprises save a certain part of the private information.

2. Research methods

In this work we, firstly, develop a theoretical game model of horizontal integration in stochastic conditions, further on we provide an analysis of the enterprises' incentives to horizontal integration and the impact the horizontally integrated companies produce on the national welfare in case the uncertainty is private information of the enterprises, then we analyze the incentives of enterprises to integration and the impact the integrated companies produce on the national welfare in case the uncertainty is public information.

The theoretical and methodological basis of this research is represented by scientific works in the area of mathematic modeling of economic processes and systems, the enterprise theories, the game theory, methodology of optimization, methods of mathematical statistics.

The following methods were used in the process of research: economic and mathematic modeling, the theory of optimization, the game theory, the theory of utilization, the theory of random functions, analysis of differential equations, comparative statistics of equilibrium.

3. Research results

We study the market of homogeneous products that are characterized by the linear demand function,

$$P(X) = a - X,$$

where a is a positive invariable, X is the consumption level. n companies are functioning in the market,

$s = 1,2,\ldots,n$, they are characterized by neutral attitude towards risk. Company production costs are determined by functions

$$C_s(x_s) = \theta_s x_s + \frac{\lambda}{2} x_s^2,$$

where x_s is the production volume, θ_s is a random parameter, while λ is a-positive constant, represented by the inverse value of the company investments. In this respect, marginal cost curves are linear and strictly rising,

$$MC_s(x_s) = \theta_s + \lambda x_s,$$

and their slope decreases, as capital investments grow. Random variables $\theta_1,\ldots,\theta_n$ are assumed as independent and distributed identically with an average $\bar{\theta}$ and variance σ_θ^2 on the interval $[\theta_{\min}, \theta_{\max}]$.

If the uncertainty is private information of the enterprises, the analysis is performed based on the following three-stage game:

1. $k(\leq n)$ companies make a decision regarding integration,
2. each company receives information regarding its own production costs, and the integrated company gets the information regarding the costs of the companies that entered the integrated structure (the insiders),
3. each company determines the production amount (the integrated company determines the production amount of the companies that entered the integrated structure).

If the uncertainty is public information, the analysis is also performed based on the following three-stage game described above, however, on the second stage each company receives information regarding the costs of all the other companies, not just its own.

Due to the symmetry principle, on the first stage it is assumed that the benefits of the joint company are equally distributed between the divisions of the integrated company. So as to avoid the phenomenon of boundary companies, i. e. when several companies are not active (i. e. produce no products), it is assumed that in order to implement all cost parameters, each company finds it to be the optimal solution to produce a positive amount of products. It means that the relative inefficiency of the companies is not so high as to result in a shutdown of the less efficient manufacturers (both inside and outside the integrated company). This condition sets certain limits on the scope of random variables regarding net demand ($a - \bar{\theta}$) for this number of companies and determines the upper variance border for random variables

$$\sigma_\theta^2 \leq \sigma_{\max}^2.$$

Let us draw up this limit.

In case integration does not take place, according to the equation (7) no companies are shut down if it is assumed that

$$x_j(\theta_{\max}) \geq 0.$$

Similarly, in case integration takes place, according to the equation (9), the outside companies are not squeezed out of the market if

$$x_o(\theta_{\max}) \geq 0,$$

whereas according to the equation (8) the integrated company will not shut down any of its enterprises under the condition that

$$x_1(\theta_{\max}, \theta_{\min}, \ldots, \theta_{\min}) \geq 0.$$

This last condition is stricter than the first two. Therefore, if the parameters meet these conditions, it is guaranteed that for any implementation of random variables and, independently of what decision on integration has been made, all companies will produce a positive amount of product. Converting this limitation and determining

$$T = \theta_{\max} - \theta_{\min} \text{ and } qT = \theta_{\max} - \theta,$$

we get the maximum length of random variable determination area

$$T_{\max}(q) = \frac{\lambda(1+\lambda)(2k+\lambda)}{[2(k-1)+q\lambda]S(n,k)}(a - \bar{\theta}),$$

$$[2(k-1)+q\lambda]S(n,k)$$

where

$$S(n,k)=(2k+\lambda)(1+\lambda)+(n-k)(k+\lambda).$$

According to the definition, maximum variance of the random variable determined on the area with the length of T, amounts to

$$\sigma^2_{\max}=q(1-q)T^2$$

where q is determined above. Substituting $T_{\max}$ in this equation and maximizing accordingly q, we get

$$\sigma^2_{\max}=\frac{\lambda^2(1+\lambda)^2(2k+\lambda)^2}{8(k-1)[2(k-1)+\lambda]S^2(n,k)}(a-\bar{\theta})^2.$$

The profit of an independent company is determined the following way

$$\pi_j=(a-X)x_j-(\theta_j+\frac{\lambda}{2}x_j)x_j \quad (1)$$

and the profit of an integration of k companies equals to the sum of profits of the companies united therein, i. e.

$$\pi_M=\sum_{i=1}^{k}\pi_i=\sum_{i=1}^{k}[(a-X)x_i-(\theta_i+\frac{\lambda}{2}x_i)x_i]. \quad (2)$$

Companies make decisions regarding the amounts of production in order to maximize the profit (on the third stage), however, the decisions regarding integration, comparing the profits expected, are made (on the first stage). Calculating the expected values in the equation (1) and in every element totalized in the equation (2), we can present the expected profits of an independent company and of companies united in an integrated structure, respectively, as two components

$$E(\pi_s)=g^D+g^U,$$

where

$$g^D=(a-\bar{X})\bar{x}_s-(\bar{\theta}+\frac{\lambda}{2}\bar{x}_s)\bar{x}_s \quad (3)$$

and

$$g^U=-E[(x_s-\bar{x}_s)(X-\bar{X})]-E[(x_s-\bar{x}_s)(\theta_s-\bar{\theta})]-\frac{\lambda}{2}E[(x_s-\bar{x}_s)^2], \quad (4)$$

where the line above represents the average value of the variable.

Function g^D determines the expected profit in an "equivalent determined market", i. e. a market with similar characteristics but without uncertainty. In an "equivalent determined market" the costs amount to $\bar{\theta}$, the amount of company's production s reach $\bar{x}_s$. And the total amount of production is $\bar{X}$. Function g^U presents additional expected profits related to existing uncertainty and the opportunity to correct the production volumes after the products are sold out. The first component of the function g^U is a negative covariance among the total production volume and the individual amount of production. As the correlation goes down, the expected profits grow. In fact, if actual individual amount of production proves to be high, a lower correlation means that the total production volume goes down and the price goes up. On the contrary, if actual individual amount of production proves to be low the price goes down. The expected price of the product goes up as the correlation goes down. The second component is a negative covariance among the production volume of a company and the border costs. The expected profit grows as the correlation goes down, for the expected costs of a company are lower. Finally, the last component is a negative variableness of individual production amounts. The expected profit grows as the variableness goes down, for the expected costs of a company are lower.

The national welfare, determined as a sum of the customers' welfare and the manufacturers' profit, can be presented as following

$$W = (a - \frac{X}{2})X - \sum_{s=1}^{n}(\theta_s + \frac{\lambda}{2}x_s)x_s .$$

Calculating the expected values, we can also define the determined and the random component in the function of expected national welfare

$$E(W) = w^D + w^U,$$

where

$$w^D = (a - \frac{\bar{X}}{2})\bar{X} - \sum_{s=1}^{n}(\bar{\theta} + \frac{\lambda}{2}\bar{x}_s)\bar{x}_s \quad (5)$$

and

$$w^U = -\frac{1}{2}E[(X - \bar{X})^2] - \sum_{s=1}^{n}E[(x_s - \bar{x}_s)(\theta_s - \bar{\theta})] - \frac{\lambda}{2}E[(x_s - \bar{x}_s)^2] . \quad (6)$$

The first component in w^U is a variance of the total production volume. The growth of total production volume variableness, when all other conditions are the same, decreases the national welfare. Though the growth of total production volume variableness leads to the growth of customers' welfare (the customers get lower prices if they consume more products), it reduces the production area profit to a significant extent. The second component is additional profits that occur due to more effective distribution of the production volume among the companies (both independent and united in an integrated structure). If the production volume of an individual company is less correlated with its variableness, it leads not only to the growth of the company profits, but also to the growth of the total national welfare. Though the growth of individual production volume variableness reduces expected profits and, therefore, the national welfare.

4. Discussing the results

4.1. Analyses of the incentives of enterprises to integration and of the impact the joint companies produce on the national welfare under the condition that the uncertainty is private information of the enterprises

In this section we make an attempt to analyze the incentives to integration and the impact the joint companies produce on the national welfare in case the uncertainty is private information of the enterprises. The first target is to determine in which market structures we can find integrated companies. The second target is to assess their influence on the national welfare.

On the third stage of the game, either all the companies function independently, or part of the companies unite to form integrated structures. In both cases, the linear-quadratic model leads to the sole Nash equilibrium. (Nash 2016; Li, Kendall & John 2016; Nachbar 2016) First, let us consider the situation when horizontal integration of enterprises does not take place. The company j chooses the production volume x_j, knowing the production costs it has θ_j, with the purpose of maximizing its profits. Within the equilibrium, the production volume x_j is determined the following way

$$x_j(\theta_j) = r_N^D(a-\bar{\theta}) - r_N^U(\theta_j - \bar{\theta}),\ j = 1,\dots,n, \quad (7)$$

where the multipliers

$$r_N^D = \frac{1}{n+\lambda+1}$$

and

$$r_N^U = \frac{1}{2+\lambda}$$

present the reaction to the determined and uncertain component of profit, respectively. For instance, $\bar{x}_j = \bar{x}_l = \bar{x}$ with any j, l. Company production volumes are lower than expected if its production costs are higher than expected and, vice versa, company production volumes are higher than expected if its production costs are lower than expected. It should be noted that the reaction of every company to utilization of random costs, its ability to react r_N^U, does not depend on the number of companies in the market. Since

random impact is independent, knowing one's own production costs does not give any additional information regarding production costs of other companies. Therefore, the company's reaction is the same and does not depend on the number of companies in the area.

Let us now consider a situation when they (k) form a horizontally integrated structure. The integrated company chooses each company production volume (i, x_i), knowing the production costs of every such company $\theta_1,\dots,\theta_k$, for the purpose of maximizing its profit (equation (2)), whereas companies-outsiders that are not part of this integrated structure, have to solve the maximizing problem we set before (equation (1)). A horizontally integrated structure produces the following product volume at every enterprise i

$$x_i(\theta_1,\dots,\theta_k) = r_I^D(a-\bar{\theta}) - r_I^{U,O}(\theta_i - \bar{\theta}) + r_I^{U,P}\sum_{p=1,p\neq i}^{k}(\theta_p - \bar{\theta}), \quad (8)$$

where $i = 1,\dots,k$, and

$$r_I^D = \frac{1+\lambda}{(2k+\lambda)(1+\lambda)+(n-k)(k+\lambda)},$$

$$r_I^{U,O} = \frac{2(k-1)+\lambda}{\lambda(2k+\lambda)},$$

$$r_I^{U,P} = \frac{2}{\lambda(2k+\lambda)}$$

present the reaction to the determined component of profit and the reaction to its own and to its partners' random impact, respectively. Information exchange gives an opportunity for a new integrated structure to rationalize production among its enterprises. In fact, every enterprise within the integrated structure increases its volume of production if the costs it bears are low $r_I^{U,O} > 0$ and if its partner bears high costs, $r_I^{U,P} > 0$. However, the reaction to its own random impact is more aggressive than the reaction to the random impacts of other enterprises,

$r_I^{U,I} < r_I^{U,O}$. Every enterprise-outsider O produces the volume of product

$$x_o(\theta_o) = r_O^D(a - \bar{\theta}) - r_O^U(\theta_o - \bar{\theta}), \quad (9)$$

where $o = k+1, \ldots, n$ and

$$r_O^D = \frac{k+\lambda}{(2k+\lambda)(1+\lambda)+(n-k)(k+\lambda)},$$

$$r_O^U = \frac{1}{2+\lambda)}.$$

Under the condition that independent companies make similar decisions on the volume of production, their expected profits are the same,

$$E(\pi_j) = E(\pi_l) = E(\pi_N)$$

at any values j, l. For the profit of an integrated structure is evenly shared among the insiders (k),

$$E(\pi_I) = \frac{E(\pi_M)}{k},$$

there is no conflict of interests, regarding preference of horizontal integration. Every company finds it profitable under the condition

$$\Delta E(\pi_I) = E(\pi_I) - E(\pi_N) \geq 0.$$

Besides, since all divisions of an integrated company make similar decisions on the volume of production, an equal share of the total expected profit is equivalent to expected profit of an individual company that is part of the integrated structure. Therefore, incentives to horizontal integration can be determined by comparing the equations (3) and (4) before and after the integration.

Therefore, incentives to horizontal integration in markets characterized by uncertainty can be analyzed by splitting the expected profit onto two components,

$$\Delta E(\pi_I) = \Delta g_I^D + \Delta g_I^U.$$

The first component Δg_I^D measures the profit changes that might have occurred in a similar determined market. From traditional point of view, integration is profitable in case the market is highly concentrated already. In fact, incentives to horizontal integration depend on the compromise between a lower volume of production ($r_I^D < r_N^D$) and higher market prices, that we get as a result of reducing the volume of production,

$$kr_I^D + (n-k)r_O^D < nr_N^D.$$

In case the market is highly concentrated (n very little), integrating companies produce a significant impact on the price and reducing the volume of production is compensated by the growth of price. For instance, integration forming a monopoly ($n = k$) is always profitable (with any λ and k). In case the market is less concentrated (capital n), integrating companies produce a smaller impact and the effect of reducing the volume of production is dominant. The second component Δg_I^U measures profit changes caused by uncertainty.

Assumption 1. In conditions of uncertainty and under the condition that the uncertainty of production costs is private information of the enterprises, companies have more incentives to horizontal integration than in a determined market. Besides, the growth of uncertainty enhances incentives to integration.

Confirmation. Substituting equations (7) and (8) in each component of equation (4), we get expected profits, related to uncertainty, before and after integration, respectively. As a result we get

$$\Delta g^U = [\frac{1}{\ldots} \ldots \frac{1}{\ldots} \ldots \frac{2(k-1)+\lambda}{\ldots} \quad \frac{1}{\ldots}$$

$$\Delta g_I^U = [-\frac{}{(2k+\lambda)^2} + \frac{}{(2+\lambda)^2} + \frac{}{\lambda(2k+\lambda)} - \frac{}{2+\lambda} -$$

$$-\frac{[2(k-1)+\lambda]^2 + 4(k-1)}{2\lambda(2k+\lambda)^2} + \frac{\lambda}{2(2+\lambda)^2}]\sigma_\theta^2 =$$

$$= [\frac{4(k-1)(1+k+\lambda)+4(k-1)}{(2+\lambda)^2(2k+\lambda)^2} + \frac{4(k-1)}{\lambda(2+\lambda)(2k+\lambda)} -$$

$$-\frac{2(k-1)[4k(1+\lambda)+\lambda(4+3\lambda)}{\lambda(2+\lambda)^2(2k+\lambda)^2}]\sigma_\theta^2 = \frac{2(k-1)}{\lambda(2+\lambda)(2k+\lambda)}\sigma_\theta^2.$$

Obviously, since $k \geq 2$, the change of profit Δg_I^U is positive and tends to grow by σ_θ^2.

Due to the fact that there is a free information exchange between the divisions of an integrated company, the integration may redistribute production from the less effective to the more effective enterprises. As shown above, the enterprise that is subject to negative random impact reacts by reducing the production volume. However, in case the company is a division of a horizontally integrated structure, the other enterprises, observing this negative impact reacts by increasing their production volume. As a result, the ineffective enterprise reduces its production still more, if it is part of this horizontally integrated structure. Likewise, in case the company, part of a horizontally integrated structure, is subject to positive random impact, it increases the production volume even more than it would being an independent unit. To sum it up, enterprises that are divisions of a horizontally integrated structure, react to market uncertainty more aggressively, $r_I^{U,O} > r_N^U$, and, therefore, the other component of equation (4) grows in case the enterprises integrate. Besides, production volume of an integrated company is less correlated with the total production volume

$$(r_I^{U,O} + (k-1)r_I^{U,P} < r_N^U$$

and, therefore, more correlated with the market price, increasing the first component of the equation (4). And though individual production of the enterprises is more volatile after integration, reducing the third component of the equation (4), this effect is always compensated by the first two.

Relevance of the stochastic component of profit against the determined one determines the incentives to horizontal integration.

Assumption 2. There are the only σ_l^2 and σ_h^2 that:

1. if $\sigma_\theta^2 \leq \sigma_l^2$, horizontal integration is profitable only in case the market is highly concentrated;
2. if $\sigma_l^2 < \sigma_\theta^2 \leq \sigma_h^2$, horizontal integration is profitable only in case the market is concentrated or highly non-concentrated;
3. if $\sigma_h^2 < \sigma_\theta^2 \leq \sigma_{\max}^2$, horizontal integration is profitable always.

Confirmation. Substituting equations (7) and (8) in equation (3), we get expected profits, at a similar determined market ($\sigma_\theta^2 = 0$) before and after integration. Subtracting the second formula from the first one, we get

$$\Delta g_I^D = \frac{1}{2}[\frac{(1+\lambda)^2(2k+\lambda)}{S^2(n,k)} - \frac{2+\lambda}{(n+\lambda+1)^2}](a-\bar{\theta})^2,$$

where

$$S(n,k) = (2k+\lambda)(1+\lambda) + (n-k)(k+\lambda).$$

This equation has the same roots n, as the equation

$$D(n) = (1+\lambda)^2(2k+\lambda)(1+\lambda+n)^2 - (2+\lambda)S(n,k)^2$$

$$D(n)=(1+\lambda)^2(2k+\lambda)(1+\lambda+n)^2-(2+\lambda)S(n,k)^2.$$

Since $D(n)$ is a quadratic function with a derivative

$$D''(n)=-2(k-1)[(2+\lambda)k+\lambda]<0$$

and

$$D(k)=(k-1)^2(1+\lambda)^2(2k+\lambda)>0,$$

there is only one value n_d, $n_d>k$, that

$$D(n_d)=0.$$

If $n\le n_d$, then

$$D(n)\ge 0 \text{ and } \Delta g_I^D(n,k)\ge 0,$$

whereas if $n>n_d$, then

$$D(n)<0 \text{ and } \Delta g_I^D(n,k)<0.$$

Thus, horizontal integration is profitable in a determined market only in case the market is highly concentrated. Obviously,

$$\lim_{n\to\infty}\Delta g_I^D(n,k)=0.$$

Let us now prove that this function has the form U by n, i. e. it has an only minimum n_m. Let us calculate the derivative

$$\frac{\partial \Delta g_I^D(n,k)}{\partial n}=-[\frac{(1+\lambda)^2(2k+\lambda)(k+\lambda)}{S^3(n,k)}-\frac{2+\lambda}{(n+\lambda+1)^3}](a-\bar{\theta})^2.$$

This function has the same roots as the function

$$P(n)=(2+\lambda)S(n,k)^3-(1+\lambda)^2(2k+\lambda)(k+\lambda)(1+\lambda+n)^3.$$

Since

$$P''(n)=6(k-1)(k+\lambda)[(2+\lambda)k+\lambda)>0$$

for all n and

$$P''(k)=6(k-1)(1+\lambda)[(2k+\lambda)k+\lambda)>0,$$

then $P''(n)$ is positive for all n and, therefore, the function $P(n)$ is convex. Since after conversion we get

$$P(k)=-(k-1)(1+\lambda)^2(2k+\lambda)[\lambda^2(2k-1)+$$

$$+\lambda(4k^2+k-1)+k(k^2+4k-1)]<0$$

and $\lim_{n\to\infty}P(n)=\infty$, there is one single value n_m that makes

$$P(n_m)=0$$

and, therefore,

$$\frac{\partial \Delta g_I^D(n_m,k)}{\partial n}=0.$$

If $n<n_m$, then

$$P(n)<0 \text{ and } \frac{\partial \Delta g_I^D(n,k)}{\partial n}<0,$$

whereas with $n\ge n_m$ we have

$$P(n)\ge 0 \text{ and } \frac{\partial \Delta g_I^D(n,k)}{\partial n}\ge 0.$$

It is obvious that

$$\Delta g_I^D(n_m, k) < 0.$$

Therefore, on the one hand, the determined curve $\Delta g_I^D(n, k)$ has the form U, where

$$\Delta g_I^D(k, k) > 0 \text{ and } \lim_{n \to \infty} \Delta g_I^D(n, k) = 0.$$

On the other hand, since the difference $\Delta g_I^U(n, k)$ is positive, does not depend on n and increases by σ_θ^2, the determined curve is shifted upwards, as uncertainty increases. Since the difference

$$\Delta g_I^U(\sigma_\theta^2)$$

increases by σ_θ^2, there are the only values σ_l^2 and σ_h^2 that make

$$\Delta g_I^D(n_{\max}, k) + \Delta g_I^U(n_{\max}, k, \sigma_l^2) = 0$$

and

$$\Delta g_I^D(n_m, k) + \Delta g_I^U(n_m, k, \sigma_h^2) = 0,$$

where $n_{\max}$ is the greatest fragmented area, and n_m is the function minimum

$$\Delta g_I^D(n, k).$$

As defined $\sigma_l^2 < \sigma_h^2$. If $\sigma_h^2 \le \sigma_{\max}^2$, there are three cases; with $\sigma_\theta^2 \le \sigma_l^2$, similarly to a determined market, there is the only value n_l, $n_l > k$, that if $n \le n_l$, then

$$\Delta g_I^D(n\ \ , k) + \Delta g_I^U(n\ \ , k, \sigma_\theta^2) \ge 0,$$

whereas with $n > n_l$

$$\Delta g_I^D(n\ \ , k) + \Delta g_I^U(n\ \ , k, \sigma_\theta^2) < 0.$$

In case

$$\sigma_l^2 < \sigma_\theta^2 \le \sigma_h^2$$

There are the only values $n_{i,1}$ and $n_{i,2}$, that if $n \le n_{i,1}$ or $n > n_{i,2}$, then

$$\Delta g_I^D(n, k) + \Delta g_I^U(n, k, \sigma_\theta^2) \ge 0,$$

whereas with

$$n_{i,1} < n \le n_{i,2},$$

then

$$\Delta g_I^D(n, k) + \Delta g_I^U(n, k, \sigma_\theta^2) < 0.$$

If $\sigma_\theta^2 > \sigma_h^2$, then

$$\Delta g_I^D(n, k) + \Delta g_I^U(n, k, \sigma_\theta^2) \ge 0$$

with all n. If

$$\sigma_l^2 \le \sigma_{\max}^2 < \sigma_h^2,$$

then the third option disappears, and if

$$\sigma_l^2 > \sigma_{\max}^2,$$

then the second and third option disappear. When we determine

$$\sigma_h^2 = \min\{\sigma_h^2, \sigma_{\max}^2\}$$

and

$$\sigma_l^2 = \min\{\sigma_l^2, \sigma_{\max}^2\}$$

we get the result of the Assumption.

At high concentration levels, the determined and the uncertain components of profit change in the same direction and, therefore, integration of enterprises is profitable. At lower concentration levels the determined component of profit goes down, whereas the uncertain component grows. If uncertainty is high, these benefits compensate for the losses of the determined components of profit. At moderate levels of uncertainty, the benefits compensate for the losses of the determined components of profit in case these losses are minor.

Assumption 3. In conditions of uncertainty and under the condition that the uncertainty of production costs is private information, companies that are not part of the integrated structure always have more advantages derived from horizontal integration.

Confirmation. Substituting equations (7) and (8) into equations (3) and (4), we get the change of expected profit for the company that is not part of the integrated structure

$$\Delta E(\pi_O) = \Delta g_O^D = \frac{2+\lambda}{2}\left[\frac{(k+\lambda)^2}{S^2(n,k)} - \frac{1}{(n+\lambda+1)^2}\right](a-\bar{\theta})^2.$$

This formula has the same roots n as the equation

$$F(n) = (k+\lambda)^2(1+\lambda+n)^2 - S(n,k)^2.$$

Since $F(n)$ is a linear function with a derivative

$$F'(n) = 2k(k-1)(k+\lambda) > 0$$

for all n and

$$F(k) = k(k-1)(k^2 + k(3+4\lambda) + 2\lambda(1+\lambda)) > 0,\ F(n) > 0,$$

then

$$\Delta g_O^D(n,k) > 0$$

with all $n \ge k$.

Though the expected volume of production of outsider enterprises is growing, the expected total volume of production goes down and, therefore, the price goes up, in case some of the enterprises integrate. Therefore, in a similar determined market profit of enterprises that are not part of the integrated structure grows. This conclusion, along with the fact that the component of expected profit related to uncertainty does not change, means that horizontal integration of enterprises in markets characterized by uncertainty always produces a positive effect on enterprises that are not part of the integrated structure. Uncertainty, however generates additional profit for the integrating companies, but not for companies-outsiders. Therefore, a great part of profit in the production area goes to the horizontally integrated enterprises.

Substituting the formula for production volume in the equations (5) and (6), we get the expected national welfare in the conditions when horizontal integration takes place and when there is no horizontal integration. The impact of horizontal integration on expected national welfare in the markets characterized by uncertainty can be presented as two components

$$\Delta W = \Delta w^D + \Delta w^U .$$

As we know, integration in determined markets without uncertainty reduces the national welfare, and therefore, this component Δw^D is negative. On the contrary, the following Assumption shows that the second component, Δw^U, is always positive.

Assumption 4. In conditions of uncertainty and under the condition that the uncertainty of production costs is private information, horizontal integration causes a less negative impact on the national welfare than in determined markets without uncertainty.

Confirmation. Substituting equations (8) and (9) into equation (6), we get the expected national welfare in case horizontal integration has taken place

$$w_M^U(n,k) = [\frac{(\lambda+3)n}{2(2+\lambda)^2} + \frac{2k(k-1)(4k+k\lambda+\lambda)}{\lambda(2+\lambda)^2(2k+\lambda)^2}]\sigma_\theta^2.$$

If horizontal integration hasn't taken place, we get

$$w_N^U(n) = w_M^U(n,1)$$

and, therefore,

$$\Delta w^U = \frac{2k(k-1)(4k+k\lambda+\lambda)}{\lambda(2+\lambda)^2(2k+\lambda)^2}\sigma_\theta^2.$$

Obviously, this function is always positive.

Horizontal integration generates profits in terms of efficiency due to information exchange among the divisions of the integrated company. In case of integration, the second component in the equation (6) grows, since the integrated enterprises produce more efficiently, but the reaction of enterprises which are not part of a horizontally integrated structure does not change. The first component of the equation (6) also grows, for volatility of the production volume of an integrated company is lower, whereas volatility of the production volume of enterprises which are not part of a horizontally integrated structure does not change. As a result, the last addend in the equation (6) is always suppressed by the first two and, therefore, the national welfare is growing.

The two previous results constitute the basis for compromise analysis of the impact horizontal integration produces on the national welfare. On the one hand, horizontal integration supports the growth of market power and the decrease of national welfare. On the other hand, horizontal integration is favorable for information exchange among divisions of an integrated company and for the growth of national welfare.

Assumption 5. In conditions of uncertainty and under the condition that the uncertainty of production costs is private information, there is the only value σ_w^2, which makes:

1. if $\sigma_\theta^2 \le \sigma_w^2$, horizontal integration always leads to decrease of national welfare;
2. if $\sigma_w^2 < \sigma_\theta^2 \le \sigma_{\max}^2$, horizontal integration leads to growth of national welfare, in case the market is moderately concentrated.

Confirmation. First, let us consider the case when k companies are integrated into one singular structure. Substituting equations (8) and (9) into equation (5), we get

$$w_M^D(n,k) = \frac{1}{2\lambda S^2(n,k)}[\sum_{r=0}^{2} v_r(k)(n-k)^r](a-\bar{\theta})^2,$$

where

$$v_0(k) = k(1+\lambda)^2(3k+\lambda),$$

$$v_1(k) = (k+\lambda)[\lambda^2 + \lambda(3k+2) + 4k],$$

$$v_2(k) = (k+\lambda)^2.$$

Next, the expected national welfare, in an equivalent determined interpretation, in case the companies decide to integrate, makes

$$w_N^D(n) = w_M^D(n,1).$$

Therefore,

$$w^D(n,k) = \frac{1}{2\lambda}\left[\frac{\sum_{r=0}^{2} v_r(k)(n-k)^r}{S^2(n,k)} - \frac{n(n+\lambda+2)}{(n+\lambda+1)^2}\right](a-\bar{\theta})^2.$$

This formula is negative if

$$G(n) = (n+\lambda+1)^2 \sum_{r=0}^{2} v_r(k)(n-k)^r - n(n+\lambda+2)S^2(n,k)$$

is negative. The function $G(n)$ is a quadratic polynomial function and its second derivative is negative

$$G''(n) = -2k(k-1)^2 < 0.$$

by virtue of the fact that

$$G'(k) = -k^2(k-1)[2\lambda^2 + \lambda(k+3) + 2] < 0$$

and

$$G(k) = -k(k-1)(1+\lambda)^2[k^2 + k(\lambda+3) + \lambda] < 0.$$

This implies that $G(n)$ and, therefore, $\Delta w^D(n,k)$ are negative at all values of $n \geq k$.

It is obvious that

$$\lim_{n\to\infty} \Delta w^D(n,k) = 0.$$

After a number of conversions we get that

$$\frac{\partial \Delta w^D}{\partial n} > 0$$

at all values $n \geq k$. Since the difference Δw^U is positive, does not depend on n and increases by σ_θ^2, we get that the determined curve is shifted upwards, as uncertainty increases. Since Δw^U increases by σ_θ^2, there can be only one singular value of σ_w^2, that makes

$$\Delta w^D(n_{\max},k) + \Delta w^U(n_{\max},k,\sigma_w^2) = 0.$$

If

$$\sigma_w^2 \leq \sigma_{\max}^2,$$

two options are possible; with

$$\sigma_\theta^2 \leq \sigma_w^2,$$

similar to the determined market,

$$\Delta w^D(n\ ,k) + \Delta w^U(n\ ,k,\sigma_\theta^2) < 0$$

with all n. With

$$\sigma_\theta^2 > \sigma_w^2$$

there is a value n_w, $n_w > k$, so that if $n \leq n_w$, then

$$\Delta g_I^D(n\ ,k) + \Delta g_I^U(n\ ,k,\sigma_\theta^2) < 0.$$

In case $n > n_w$

$$\Delta g_I^D(n, k) + \Delta g_I^U(n, k, \sigma_\theta^2) \geq 0.$$

If

$$\sigma_w^2 > \sigma_{\max}^2,$$

the second option disappears. We denote

$$\sigma_w^2 = \min\{\sigma_w^2, \sigma_{\max}^2\}$$

and get the result of the Assumption.

In case the uncertainty is low with respect to demand, loss of market power exceeds informational profits and, therefore, the expected national welfare always goes down. In case the uncertainty is high, informational profits exceed loss of market power if the latter are not large.

4.2. Analyses of the incentives of enterprises to integration and of the impact the joint companies produce on the national welfare under the condition that the uncertainty is public information of the enterprises

In the previous section, we showed that uncertainty of market conditions increases the incentives of enterprises to integration and reduces the impact of horizontal integration on the national welfare. These results are due to information exchange among divisions of an integrated company and consequent streamlining of production output among integrated enterprises. In this section, we determine that information exchange among the enterprises of an integrated company is crucial for such conclusions. In this section, we show that in conditions when the uncertainty of company production costs is public information, the companies may have less incentives to integration and horizontally integrated companies may be less efficient than in determined conditions. In this case, horizontal integration does not contribute to the aggregation of information, for the production costs of divisions within an integrated company are common knowledge even without any information leaks.

The next Assumption compares incentives to integration under the condition when uncertainty of company production costs is public information, with the situation when uncertainty of production costs is private information of enterprises and when there is no uncertainty as such.

Assumption 6. In the uncertain market reality, companies have less incentives to integration in conditions when uncertainty of company production costs is public information, than when uncertainty of production costs is private information of enterprises. Besides, in the uncertain market reality and under the condition that uncertainty of production costs is public information:

1. if the number of integrated companies is small and the original number of companies in the production field is large, companies have less incentives to integration than in determined markets;

2. if the number of integrated companies is large and the original number of companies in the production field is small, companies have more incentives to integration than in determined markets.

Confirmation. In case integration took place on the first stage, determining

$$\tilde{\theta} = \frac{1}{n}\sum_{i=1}^{n}\theta_i,$$

we get that the production output of each enterprise that joined the integrated structure will amount to

$$x_i(\theta_1, \dots, \theta_n) = r_I^{D,F}(a - \tilde{\theta}) - r_{I,O}^{U,F}(\theta_i - \tilde{\theta}) + r_{I,P}^{U,F} \sum^{k}(\theta_n - \tilde{\theta}),$$

$p=1,p\neq i$

where

$$r_I^{D,F}=r_I^D,$$

$$r_{I,O}^{U,F}=\frac{(\lambda+k-1)S+\lambda(k-1)}{\lambda(k+\lambda)S},$$

$$r_{I,P}^{U,F}=\frac{S-\lambda(k-1)}{\lambda(2+\lambda)S},$$

and the production output of each enterprise that has not joined the integrated structure will amount to

$$x_o(\theta_1,...,\theta_n)=r_O^{D,F}(a-\widetilde{\theta})-r_{O,O}^{U,F}(\theta_o-\widetilde{\theta})+r_{O,P}^{U,F}\sum_{p=k+1,p\neq o}^{n}(\theta_p-\widetilde{\theta}),$$

where

$$r_O^{D,F}=r_O^D,\ r_{O,O}^{U,F}=\frac{S-(k-1)}{(1+\lambda)S},\ r_{O,P}^{U,F}=\frac{k-1}{(1+\lambda)S}.$$

Substituting $k=1$, into these formulas, we get the result for the option when on the first stage integration didn't take place. Expected production volume in conditions when integration took or didn't take place, are the same as in the situation when uncertainty of production costs is private information of enterprises.
We can express the expected profit in two components:

$$E(\pi^F)=g^{D,F}+g^{U,F}.$$

On the one hand, since the expected production volume is the same as in conditions when uncertainty of production costs is private information of enterprises, the first component will be the same as in conditions when uncertainty of production costs is private information of enterprises, $g^{D,F}=g^D$. Let us calculate the expected profit in conditions of uncertainty for the enterprises that are part of the integrated structure:

$$g_I^{U,F}=\frac{1}{2\lambda S^2}[\sum_{r=0}^{2}\varphi_r(k)(n-k)^r]\sigma_\theta^2,$$

where

$$\varphi_0(k)=(1+\lambda)^2(2k+\lambda)(2(k-1)+\lambda),$$

$$\varphi_1(k)=(2k+\lambda)[2(k-1)(1+\lambda)+\lambda(2\lambda+3)],$$

$$\varphi_2(k)=(k-1)(k+\lambda)+\lambda(k+1+\lambda).$$

Calculating the difference, we get

$$g_I^{U,F}(n,k) - g_I^U(n,k) = \frac{1}{2(2k+\lambda)S^2}[\sum_{r=1}^{2}\phi_r(k)(n-k)^r]\sigma_\theta^2,$$

where

$$\phi_1(k) = (2k+\lambda)[2(k+1)+3\lambda),$$

$$\phi_2(k) = 3k + 2\lambda .$$

Obviously, this difference is positive. Therefore, profit in conditions of uncertainty is higher for the enterprises that are part of the integrated structure and the outsiders (with $k = 1$).
We have

$$g_I^U - g_I^{U,F} = (g_N^{U,F} - g_N^U) - (g_I^{U,F} - g_I^U).$$

This difference is positive if the formula

$$J(n) = (2k+\lambda)S^2(n,k)[\sum_{r=1}^{2} \phi_r(1)(n-1)r - (2+\lambda)S^2(n,1)[\sum_{r=1}^{2} \phi_r(n-k)r$$

is positive. $J(n)$ is a quartic polynomial function. At that, $J^{(4)}(n) > 0$ with all n, and besides

$$J'''(k) > 0, J''(k) > 0, J'(k) > 0, J\ (k) > 0.$$

Therefore $J(n) > 0$ and, consequently,

$$\Delta g_I^D - \Delta g_I^{U,F} > 0.$$

As a result we get

$$g_I^{U,F}(n,k) = \frac{k-1}{2\lambda(1+\lambda)^2(n+\lambda+1)^2 S^2}[\sum_{r=0}^{4} \xi_r(k)(n-k)^r]\sigma_\theta^2,$$

where

$$\xi_0(k) = (1+\lambda)^2(2k+\lambda)(\lambda^3 + 6\lambda^2 + \lambda(5k+7) + 2(k+1)^2),$$
$$\xi_1(k) = (1+\lambda)(2k+\lambda)(k+2\lambda+1)(\lambda^2 + 7\lambda + 2k + 6),$$
$$\xi_2(k) = 2\lambda^3(k+5) + 2\lambda^2(15k+8) + 2\lambda(6k^2 + 20k + 3) + k(k^2 + 10k + 13),$$
$$\xi_3(k) = -2\lambda^3 - \lambda^2(k-1) + \lambda(8k+2) + 2k(k+3),$$
$$\xi_4(k) = -\lambda^2 + k .$$

The difference $g_I^{U,F}$ is positive if the second multiplier is positive. Functions $\xi_0(k)$, $\xi_1(k)$, $\xi_2(k)$ are positive but $\xi_3(k)$ and $\xi_4(k)$ may be negative. The second derivative of

$$\sum_{r=0}^{4} \xi_r(k)(n-k)^r$$

equals to

$$12\xi_4(k)(n-k)^2 + 6\xi_3(k)(n-k) + \xi_2(k)$$

and is positive as $n = k$.
Let us assume that

$$k < k^* = \lambda^2 .$$

Then $\xi_4(k)$ is negative and the second derivative of the sum is a concave function. Thus, there is one singular value n^*, such that if $n > n^*$, the sum and, consequently, $g_I^{U,F}$, are negative, while with $n \le n^*$ these functions are positive. If, on the other hand, $k \ge k^*$, then $\xi_4(k)$ is not negative.

If $\xi_3(k)$ is also positive, the sum and $g_I^{U,F}$ will be positive with all n. However, $\xi_3(k,\lambda)$ is a concave function by λ, at that,

$$\xi_3(k,0) > 0 \text{ and } \xi_3(k,\sqrt{k}) > 0$$

and, therefore, with $\lambda \leq \sqrt{k}$ the function $\xi_3(k,\lambda)$ is positive.

In the conditions when uncertainty of company production costs is public information, streamlining of production output among horizontally integrated enterprises is not so strong as in the conditions when uncertainty of company production costs is private information. First, integrating companies would be responsible for utilizing the costs of their partners even if they are not part of the integrated system. Second, enterprises that are not part of the integrated system can react to utilization of costs of each integrated company in this particular situation. As a result, variation of the stochastic component of the expected profit is lower than in conditions when uncertainty of company production costs is private information. The difference may even be negative which may make horizontally integrated companies less profitable than in determined markets. In fact, integrating companies face a compromise between less aggressive behavior and higher price. In the long run, whether uncertainty leads to profits or loss again depends on the relative percentage of enterprises that join an integrated structure. In case this percentage covers a significant part of the market, integrating companies benefit from the uncertainty present at the market, whereas if this percentage covers an insignificant part of the market, integrating companies bear losses from the uncertainty present at the market. In the following Assumption we show that the latter phenomenon occurs only when the enterprises have no incentives to integration in determined circumstances.

Assumption 7. In conditions when uncertainty of company production costs is public information, more integrated structures tend to appear than in determined market conditions.

Confirmation. We need to prove that if the difference $\Delta g_I^{U,F}$ is negative, the difference Δg_I^D is negative, too. Under the condition that both functions are positive with $n=k$ and $\Delta g_I^D(n)$ traverses the horizontal axis only once, and $\Delta g_I^{U,F}(n)$ traverses the horizontal axis only once or does not traverse at all, it is sufficient to show that $\Delta g_I^{U,F}(n)$ are positive at the intersection point of functions Δg_I^D,

$$\Delta g_I^{U,F}(n^d) > 0,$$

where the point n^d is determined as the root of the equation

$$\Delta g_I^D(n^d) = 0.$$

After substitution we get

$$g_I^{U,F}(n^d,k) = \frac{k-1}{2\lambda(n^d+\lambda+1)^2(1+\lambda)^2 S^2}[\sum_{r=0}^{2}\chi_r(k)(n^d-k)^r]\sigma_\theta^2,$$

where

$$\chi_0(k) = (2k+\lambda)(2-\lambda(k-2)),$$
$$\chi_1(k) = 2(2k+\lambda),\ \chi_2(k) = k.$$

After convertion we get

$$\Delta g_I^D(n,k) = \frac{k-1}{2\lambda(n^d+\lambda+1)^2 S^2}[\sum_{r=0}^{2}\tau_r(k)(n-k)^r](a-\bar{\theta})^2,$$

where

$$\tau_0(k) = (k-1)(1+\lambda)^2(2k+\lambda),$$
$$\tau_1(k) = -2(1+\lambda)(2k+\lambda),$$
$$\tau_2(k) = -(k(2+\lambda)+\lambda).$$

Then

$$n^d = k - \frac{\tau_1 + \sqrt{(\tau_1)^2 - 4\tau_0\tau_2}}{2\tau_2}.$$

Since

$$\frac{-\chi_1 + \sqrt{(\chi_1)^2 - 4\chi_0\chi_2}}{2\chi_2} \geq -\frac{\tau_1 + \sqrt{(\tau_1)^2 - 4\tau_0\tau_2}}{2\tau_2},$$

$n^d \geq n^c$, where

$$n^c = k + \frac{-\chi_1 + \sqrt{(\chi_1)^2 - 4\chi_0\chi_2}}{2\chi_2}.$$

For a fixed number of integrating companies, both the determined and the random components of expected profit are negative in case the initial number of companies in the market (in the production area) is big. In the presence of a big number of enterprises that are not part of the integrated structure, the rundown of production volume for integrating enterprises is not compensated by a moderate rise of price. The problem, however, is much more serious in the determined sector of expected profit than it is in the random component of expected profit. As expected, a horizontally integrated company reduces the production volume evenly among the divisions, however, de facto, the integrated company can distribute the rundown of production volume more efficiently.

Assumption 8. In the uncertain market reality, horizontal integration produces a significantly more negative impact on the national welfare in conditions when uncertainty of company production costs is public information, than when uncertainty of production costs is private information of enterprises. Besides, in the uncertain market reality in case uncertainty is public information:

1. in case the number of companies that join an integrated structure is insignificant and the initial number of companies in the area is great, horizontally integrated companies produce a significantly more negative impact on the national welfare than in determined markets;

2. in case the number of companies that join an integrated structure is significant and the initial number of companies in the area is small, horizontally integrated companies produce a significantly less negative impact on the national welfare than in determined markets.

Confirmation. Suppose that

$$E(W^F) = w^{D,F} + w^{U,F}.$$

We have $w^{D,F} = w^D$ and

$$w_M^{U,F}(n,k) = \frac{1}{2\lambda(1+\lambda)^2 S^2}\left[\sum_{r=0}^{2}\eta_r(k)(n-k)^r\right]\sigma_\theta^2,$$

where

$$\eta_0(k) = k(1+\lambda)4(4k^2 + 4(\lambda-1)k + \lambda(\lambda-1)),$$

$$\eta_1(k) = (1+\lambda)[4(1+\lambda)^2 k^3 + (10\lambda^3 + 19\lambda^2 + \lambda - 4)k^2 + 6\lambda^3(\lambda+2)k +$$

$$+\lambda^3(\lambda^2 + 2\lambda - 1)],$$

$$\eta_2(k) = (1+\lambda^2)k^3 + (6\lambda^3 + 14\lambda^2 + 6\lambda - 1)k^2 + \lambda^2(7\lambda^2 + 18\lambda + 9)k +$$

$$+(2\lambda^2 + 5\lambda + 2)\lambda^3,$$

$$\eta_3(k) = \lambda(2+\lambda)(k+\lambda)^2.$$

Hence, we get that the difference

$$\Delta w^U - \Delta w^{U,F} \geq 0.$$

Besides, it can be proven that

$$\Delta w^U - \Delta w^D < 0$$

if and only if $k \leq k^*$ and $n > n^*$.

In volatile markets, horizontally integrated companies are not only less profitable, in case uncertainty of company production costs is public information, but also lead to decrease of national welfare. In case uncertainty is public information, variability of the random component of the expected national welfare is lower than if uncertainty is private information, since, as it was stated above, streamlining of production among enterprises is less efficient. This difference may even be negative, which proves lower efficiency of horizontal integration in these conditions than in a similar determined market. Though production is more efficiently performed, the variance may grow. A higher variance increases the consumers' welfare, however, it reduces the total national welfare. In the long run, the impact produced by horizontal integration on the national welfare depends on the relevant significance of the determined and stochastic components of profit.

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5. Conclusion

In the present work we analyzed the impact of uncertainty and private information on the horizontal integration in production area (in the market). According to conventional assumptions, enterprises have more incentives to integration in areas (markets) characterized by a higher degree of uncertainty. Formal analysis that was done in this work shows that this is not always the case. If the uncertainty in the area (in the market) is public information, the enterprises may have less incentives to integration than in determined markets. On the contrary, if the uncertainty in the area (in the market) is private information, the enterprises may have more incentives to integration than in determined markets.

The results when the uncertainty in the area (in the market) is private information have been achieved under the condition that the enterprises receive ideal signals regarding their uncertain and independent characteristics. The enterprises also have more incentives to integration in case these characteristics are interrelated. However, in case these characteristics are correlated to a large extent, the enterprises can have more incentives to integration than in determined markets. Actually, in this case, the enterprises can estimate all the information on their competitors and, therefore, the situation is close to the one when the uncertainty in the area (in the market) is public information. Consequently, integration in the markets characterized by uncertainty is more profitable in case the enterprises save a certain part of the private information.

The impact of horizontal integration on the national welfare in the markets characterized by uncertainty also depends on the type of information. In case the uncertainty in the area (in the market) is public information, horizontally integrated companies can produce a more negative impact on the national welfare than when they act in determined markets. In case the uncertainty in the area (in the market) is private information, horizontally integrated companies are a more frequent phenomenon and are more favorable for the national welfare.

In markets characterized by high volatility these benefits can compensate for the anti-competitor effects of horizontal integration and, therefore, horizontally integrated companies, in case the uncertainty in the area (in the market) is private information, increase the national welfare.

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